1. In Figure P.1, the cube is 40.0 cm on each edge. Four straight segments of wire—ab, bc, cd, and da—form a closed loop that carries a current \( I = 5.00 \, \text{A} \), in the direction shown. A uniform magnetic field of magnitude \( B = 0.0200 \, \text{T} \) is in the positive y direction. Determine the magnitude and direction of the magnetic force on each segment.

![Diagram of a cube with segments ab, bc, cd, da forming a closed loop with a current I, and a magnetic field B in the positive y direction.](image)

2. A nonconducting sphere has mass 80.0 g and radius 20.0 cm. A flat compact coil of wire with 5 turns is wrapped tightly around it, with each turn concentric with the sphere. As shown in Figure P2, the sphere is placed on an inclined plane that slopes downward to the left, making an angle \( \theta \) with the horizontal, so that the coil is parallel to the inclined plane. A uniform magnetic field of 0.350 T vertically upward exists in the region of the sphere. What current in the coil will enable the sphere to rest in equilibrium on the inclined plane? Show that the result does not depend on the value of \( \theta \).

![Diagram of a sphere with a coil wrapped around it, placed on an inclined plane with a magnetic field B vertically upward.](image)

3. Two circular coils of radius \( R \), each with \( N \) turns, are perpendicular to a common axis. The coil centers are a distance \( R \) apart. Each coil carries a steady current \( I \) in the same direction, as shown in Figure P3. (a) Show that the magnetic field on the axis at a distance \( x \) from the center of one coil is

\[
B = \frac{N\mu_0 I R^2}{2} \left[ \frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2Rx)^{3/2}} \right]
\]

(b) Show that \( dB/dx \) and \( d^2 B/dx^2 \) are both zero at the point midway between the
coils. This means the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called Helmholtz coils.

4. (from Halliday, problem 29.28, page 785)
In Fig. 1, part of a long insulated wire carrying current \( i = 5.78 \text{ mA} \) is bent into a circular section of radius \( R = 1.89 \text{ cm} \). In unit-vector notation, what is the magnetic field at the center of curvature \( C \) if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated?

**Solution:**
(a) The contribution to \( B_C \) from the (infinite) straight segment of the wire is

\[
B_{C1} = \frac{\mu_0 i}{2\pi R}.
\]

The contribution from the circular loop is \( B_{C2} = \frac{\mu_0 i}{2R} \). Thus,

\[
B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R} \left( 1 + \frac{1}{\pi} \right) = \frac{\left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 5.78 \times 10^{-3} \text{ A} \right)}{2 \left( 0.0189 \text{ m} \right)} \left( 1 + \frac{1}{\pi} \right) = 2.53 \times 10^{-7} \text{T}.
\]

\( \vec{B}_C \) points out of the page, or in the \( +z \) direction. In unit-vector notation,

\[
\vec{B}_C = (2.53 \times 10^{-7} \text{T}) \hat{k}
\]

(b) Now \( \vec{B}_{C1} \perp \vec{B}_{C2} \) so

\[
B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R} \sqrt{1 + \frac{1}{\pi^2}} = \frac{\left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 5.78 \times 10^{-3} \text{ A} \right)}{2 \left( 0.0189 \text{ m} \right)} \sqrt{1 + \frac{1}{\pi^2}} = 2.02 \times 10^{-7} \text{T}.
\]
and \( \vec{B}_c \) points at an angle (relative to the plane of the paper) equal to
\[
\tan^{-1}\left( \frac{B_{c1}}{B_{c2}} \right) = \tan^{-1}\left( \frac{1}{\pi} \right) = 17.66^\circ.
\]

In unit-vector notation,
\[
\vec{B}_c = 2.02 \times 10^{-7} \text{T}(\cos 17.66^\circ \hat{i} + \sin 17.66^\circ \hat{k}) = (1.92 \times 10^{-7} \text{T})\hat{i} + (6.12 \times 10^{-8} \text{T})\hat{k}
\]

5. (from Halliday, problem 29.32, page 785)

Figure 2 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm. Wire 1 carries current \( i_1 = 60.0 \) mA out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current \( i_2 = 40.0 \) mA out of the page and can be moved around the cylinder. At what (positive) angle \( \theta_2 \) should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT?

\[\text{Fig. 2.}\]

Solution:

By the right-hand rule (which is “built-into” Eq. 29-3) the field caused by wire 1’s current, evaluated at the coordinate origin, is along the +y axis. Its magnitude \( B_1 \) is given by Eq. 29-4. The field caused by wire 2’s current will generally have both an x and a y component which are related to its magnitude \( B_2 \) (given by Eq. 29-4) and sines and cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle \( \theta_2 \) (shown in Fig. 29-60) then its components are
\[
B_{2x} = B_2 \sin \theta_2, \quad B_{2y} = -B_2 \cos \theta_2.
\]

The magnitude-squared of their net field is then (by Pythagoras’ theorem) the sum of the square of their net x-component and the square of their net y-component:
\[
B^2 = (B_{2x})^2 + (B_{2y}^2 - B_1^2 - 2B_1 B_2 \cos \theta_2)^2 = B_1^2 + B_2^2 - 2B_1 B_2 \cos \theta_2.
\]

(since \( \sin^2 \theta + \cos^2 \theta = 1 \)), which we could also have gotten directly by using the law of
cosines. We have

\[ B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}. \]

With the requirement that the net field have magnitude \( B = 80 \text{ nT} \), we find

\[ \theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1B_2}\right) = \cos^{-1}\left(-\frac{1}{4}\right) = 104^\circ, \]

where the positive value has been chosen.

6. (from Halliday, problem 29.58, page 787)
Figure 3 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius \( R = 25.0 \text{ cm} \), separated by a distance \( s = R \). The two coils carry equal currents \( i = 12.2 \text{ mA} \) in the same direction. Find the magnitude of the net magnetic field at \( P \), midway between the coils.

![Fig. 3.](image)

7. (from Halliday, problem 29.62, page 788)
In Fig. 4a, two circular loops, with different currents but the same radius of 4.0 cm, are centered on a \( y \) axis. They are initially separated by distance \( L = 3.0 \text{ cm} \), with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with \( y \) component \( B_y \). That component is to be measured as loop 2 is gradually moved in the positive direction of the \( y \) axis. Figure 4b gives \( B_y \) as a function of the position \( y \) of loop 2. The curve approaches an asymptote of \( B_y = 7.20 \mu\text{T} \) as \( y \to \infty \). The horizontal scale is set by \( y_s = 10.0 \text{ cm} \). What are (a) current \( i_1 \) in loop 1 and (b) current \( i_2 \) in loop 2?
Fig. 1.
Solution:
Using Halliday Eq. 29-26, we find that the net y-component field is

\[ B_y = \frac{\mu_0 i_1 R_1^2}{2\pi(R^2 + z_1^2)^{3/2}} - \frac{\mu_0 i_2 R_2^2}{2\pi(R^2 + z_2^2)^{3/2}}, \]

where \( z_1^2 = L^2 \) (see Fig. 1(a)) and \( z_2^2 = y^2 \) (because the central axis here is denoted \( y \) instead of \( z \)). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 1(b) corresponding to \( B_y = 0 \) would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As \( y \to \infty \), only the first term contributes and (with \( B_y = 7.2 \times 10^{-6} \text{T} \)) we can solve for \( i_1 \). We obtain \( i_1 = \left(\frac{45}{16\pi}\right) \approx 0.90 \text{ A} \).

(b) With loop 2 at \( y = 0.06 \text{ m} \) (see Fig. 1(b)) we are able to determine \( i_2 \) from

\[ \frac{\mu_0 i_1 R^2}{2(R^2 + L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2 + y^2)^{3/2}}. \]

We obtain \( i_2 = (117\sqrt{13}/50\pi) \approx 2.7 \text{ A} \).

8. (from Halliday, problem 29.65, page 788)
Figure 5 shows a cross section of a long cylindrical conductor of radius \( a = 4.00 \text{ cm} \) containing a long cylindrical hole of radius \( b = 1.50 \text{ cm} \). The central axes of the cylinder and hole are parallel and are distance \( d = 2.00 \text{ cm} \) apart; current \( i = 5.25 \text{ A} \) is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases \( b = 0 \) and \( d = 0 \).
Solution:
(a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current $i$ which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance $r$ from its axis, inside the cylinder. Here $R$ is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance $r_1$ from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2 (a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)}.$$
and the field it produces at a point inside, a distance \( r_2 \) from the its axis, has magnitude

\[
B_2 = \frac{\mu_0 I_2 r}{2\pi b^2} = \frac{\mu_0 i r^2}{2\pi b^2 (a^2 - b^2)} = \frac{\mu_0 i r^2}{2\pi (a^2 - b^2)}.
\]

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place \( r_1 = d \) in the expression for \( B_1 \) and obtain

\[
B = \frac{\mu_0 i d}{2\pi (a^2 - b^2)} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (5.25 \text{ A}) (0.0200 \text{ m})}{2\pi [(0.0400 \text{ m})^2 - (0.0150 \text{ m})^2]} = 1.53 \times 10^{-5} \text{ T}
\]

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If \( b = 0 \) the formula for the field becomes

\[
B = \frac{\mu_0 i d}{2\pi a^2}.
\]

This correctly gives the field of a solid cylinder carrying a uniform current \( i \), at a point inside the cylinder a distance \( d \) from the axis. If \( d = 0 \) the formula gives \( B = 0 \). This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

Note: One might apply Ampere’s law to show that the magnetic field in the hole is uniform. Consider a rectangular path with two long sides (side 1 and 2, each with length \( L \)) and two short sides (each of length less than \( b \)). If side 1 is directly along the axis of the hole, then side 2 would be also parallel to it and also in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere’s law, we might wish to make \( L \) very long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between \( \vec{B} \) and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere’s law reduces to

\[
\oint_{\text{rectangle}} \vec{B} \cdot d\vec{s} = \mu_0 i
\]

\[
\int_{\text{side 1}} \vec{B} \cdot d\vec{s} + \int_{\text{side 2}} \vec{B} \cdot d\vec{s} = \mu_0 i \text{ enclosed}
\]

\[
(B_{\text{side 1}} - B_{\text{side 2}}) L = 0
\]

where \( B_{\text{side 1}} \) is the field along the axis found in part (a). This shows that the field at off-axis points (where \( B_{\text{side 2}} \) is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

9. (from Halliday, problem 29.81, page 789)

Figure 6 shows a cross section of a hollow cylindrical conductor of radii \( a \) and \( b \), carrying a
uniformly distributed current \( i \). (a) Show that the magnetic field magnitude \( B(r) \) for the radial distance \( r \) in the range \( b < r < a \) is given by

\[
B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \frac{r^2 - b^2}{r}.
\]

(b) Show that when \( r = a \), this equation gives the magnetic field magnitude \( B \) at the surface of a long straight wire carrying current \( i \); when \( r = b \), it gives zero magnetic field; and when \( b = 0 \), it gives the magnetic field inside a solid conductor of radius \( a \) carrying current \( i \).

Fig. 2.

Solution:
(a) For the circular path \( L \) of radius \( r \) concentric with the conductor

\[
\oint_L \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}.
\]

Thus, \( B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \left( \frac{r^2 - b^2}{r} \right) \).

(b) At \( r = a \), the magnetic field strength is

\[
\frac{\mu_0 i}{2\pi(a^2 - b^2)} \left( \frac{a^2 - b^2}{a} \right) = \frac{\mu_0 i}{2\pi a}.
\]

At \( r = b, B \propto r^2 - b^2 = 0 \). Finally, for \( b = 0 \)

\[
B = \frac{\mu_0 i}{2\pi a^2} \frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}
\]

which agrees with Eq. 29-20.