國立中山大學 101 學年度普通物理(一)期中考　　101.11.21

答題須知：

一、請由答題紙第六列開始填寫，並標明題號。考試時間為 19：00~21：30，20：00 以後方可交卷。

二、請詳列相關公式及計算過程，記得寫上單位，建議以M.K.S制表示。遇力學題目時，請畫出力學圖解圖，未畫者將扣分。

1. (10%) Given the vectors $\vec{A}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=\hat{i}-\hat{j}+\hat{k}$.

   (a) Find the angles between $\vec{A}$ and *x*, *y*, *z* coordinates axes.

   (b) Find the unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$.

(a) $\because\ \vec{A}=2\hat{i}+\hat{j}-2\hat{k}\quad \therefore A=[2^2+1^2+(-2)^2]^{1/2}=3$ 【1%】

設α, β, γ 為 $\vec{A}$ 與 x, y, z 軸之夾角

$\vec{A}\cdot\hat{i}=2=|\vec{A}||\hat{i}|\cos\alpha \quad \cos\alpha=\frac{2}{3} \quad \alpha=\cos^{-1}\frac{2}{3}$ 【1%】

$\vec{A}\cdot\hat{j}=1=|\vec{A}||\hat{j}|\cos\beta \quad \cos\beta=\frac{1}{3} \quad \beta=\cos^{-1}\frac{1}{3}$ 【1%】

$\vec{A}\cdot\hat{k}=-2=|\vec{A}||\hat{k}|\cos\gamma \quad \cos\gamma=\frac{-2}{3} \quad \gamma=\cos^{-1}\frac{-2}{3}$ 【1%】

(b) $\hat{n}=\pm\frac{(\vec{A}\times\vec{B})}{|\vec{A}\times\vec{B}|}$ 【2%】

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & -1 & 1\end{vmatrix}=-\hat{i}-4\hat{j}-3\hat{k}$$

$|\vec{A}\times\vec{B}|=[(-1)^2+(-4)^2+(-3)^2]^{1/2}=26^{1/2}$ 【2%】

$\therefore\ \hat{n}=\pm\frac{1}{\sqrt{26}}(-\hat{i}-4\hat{j}-3\hat{k})$ 【2%】

2. (a) (5%) In Fig. 1, a gun is mounted on a cliff at height $\boldsymbol{h}$ above the ground. It fires a projectile with velocity $\boldsymbol{v_0}$ at an elevation angle $\boldsymbol{\alpha}$. Find the horizontal range $\boldsymbol{R}$.

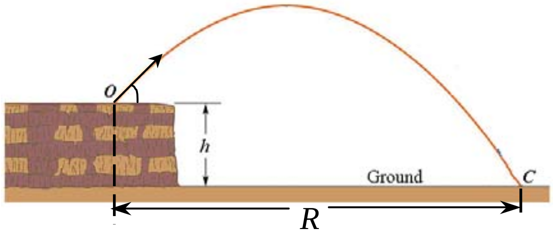


Fig. 1

(b) (5%) In (a), we find that R has maximum range when α is equal to $\sin^{-1}\dfrac{v_0}{[2(v_0^2+gh)]^{1/2}}$. Show that the flying time of the projectile from point O to C for this angle is $\dfrac{v_0}{g\sin\alpha}$.

(a) 設物體由 O 飛行至 C 點之時間爲 t，取 O 爲原點，則

$$-h = v_0 \sin\alpha\, t - \frac{1}{2}gt^2 \qquad \therefore gt^2 - 2v_0\sin\alpha\, t - 2h = 0$$

$$\therefore t = \frac{2v_0\sin\alpha \pm [4v_0^2\sin^2\alpha + 8gh]^{1/2}}{2g} \qquad (t\ 取\oplus值)$$

$$\therefore t = \frac{1}{g}[v_0\sin\alpha + (v_0^2\sin^2\alpha + 2gh)^{1/2}]$$

$$\therefore R = (v_0\cos\alpha)t = \frac{v_0\cos\alpha}{g}[v_0\sin\alpha + (v_0^2\sin^2\alpha + 2gh)^{1/2}]$$

(b) $\because \sin\alpha = \dfrac{v_0}{[2(v_0^2+gh)]^{1/2}}$

$$\therefore t = \frac{1}{g}[v_0\frac{v_0}{[2(v_0^2+gh)]^{1/2}} + (\frac{v_0^4}{[2(v_0^2+gh)]} + 2gh)^{1/2}]$$

$$= \frac{1}{g}[\frac{v_0^2}{[2(v_0^2+gh)]^{1/2}} + \frac{v_0^2+2gh}{[2(v_0^2+gh)]^{1/2}}] = \frac{1}{g}[2(v_0^2+gh)]^{1/2} = \frac{v_0}{g\sin\alpha}$$

3. (5%) In Fig. 2, a block of mass $\boldsymbol{M}$ slides along a floor while a constant force $\vec{F}$ is applied at an upward angle $\boldsymbol{\theta}$. The coefficient of kinetic friction between the block and the floor is μ_k. We can vary θ from 0 to 90°, what θ gives the maximum value of the block's acceleration magnitude?

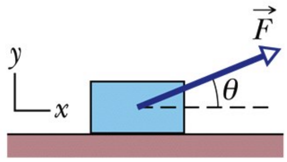


Fig. 2

$F\cos\theta - f_k = Ma$ (1) 【1%】

$F_N = Mg - F\sin\theta$ (2) 【1%】

$f_k = \mu_k F_N$ (3) 【1%】

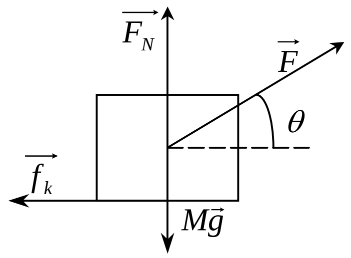
由(1)(2)(3)

$F\cos\theta - \mu_k (Mg - F\sin\theta) = Ma$

$\therefore a = \dfrac{F}{M}\cos\theta - \mu_k (g - \dfrac{F}{M}\sin\theta)$ (4)

令 $\dfrac{da}{d\theta} = 0 \quad \therefore -\dfrac{F}{M}\sin\theta + \mu_k \dfrac{F}{M}\cos\theta = 0$

$\therefore \tan\theta = \mu_k \quad \Rightarrow \quad \theta = \tan^{-1}\mu_k$ 【2%】



4. An elevator cab of mass $\boldsymbol{m}$ **= 400 kg** (Fig. 3) is descending with initial speed $\boldsymbol{v_i}$ **= 5.0 m/s** when its supporting cable begins to slip, allowing it to fall with constant acceleration $\boldsymbol{a = g/4}$.
   (a) (10%)During the fall through a distance $\boldsymbol{d}$ **= 10 m**, what is the **net** work done on the cab? (Please draw a simple figure showing all possible forces applied on the cab.)
   (b) (5%) Use "Work-Kinetic Energy Theorem" to obtain the final speed of the cab.

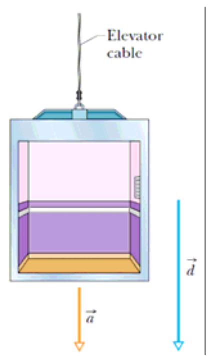
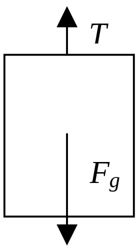


Fig. 3

(a)
Simple figure:



【4%】

The net force eventually leads to a downward acceleration $a = g/4$.
Thus, the net work done on the cab is
$W_{net} = m \cdot a \cdot d = 400 \cdot (9.8/4) \cdot 10 = 9800$ (J) 【6%】
(One can also calculate the work done by the gravitational force and the tension separately. In other words, $W_g = mgd$ =39200 (J) and $W_T = -3mgd/4 = -29400$ (J); the net work done is just their sum.)

(b)
Work-Kinetic Energy Theorem: $\Delta K = W$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$ 【2%】

$$v_f = \sqrt{\frac{2}{m}(W + \frac{1}{2}mv_i^2)} = \sqrt{\frac{2}{400}(9800 + \frac{1}{2}400 \cdot 5^2)} = 8.6\ \text{(m/s)}$$ 【3%】

5. A **10.0 kg** breadbox on a frictionless incline of angle θ is connected, by a cord that runs over a pulley, to a light spring of spring constant $\boldsymbol{k}$ **= 340 N/m**, as shown in Fig. 4. The box is released from rest when the spring is unstretched. The speed of the box reaches **0.8 m/s** when it has moved **10 cm** down the incline. Assume that the pulley is massless and frictionless.

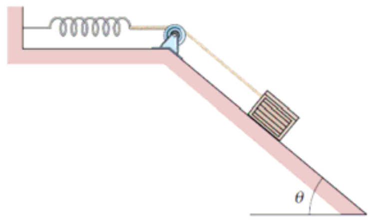


Fig. 4

   (a) (5%)What is the incline angle θ?
   (b) (5%)How far down the incline from its point of release does the box slide before momentarily stopping?

(a)

The loss of gravitational potential energy becomes the gain of kinetic energy + elastic potential energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$ 【2%】

$(10.0)(9.8)(0.1\sin\theta) = (0.5)(10.0)(0.8)^2+(0.5)(340)(0.1)^2$

$\therefore\ \sin\theta = 1/2,\quad \theta = 30^{\circ}$ 【3%】

(b)

The gravitational potential energy loss now totally converts into the elastic one.

Thus, assuming such farthest distance is d

$$mgh = \frac{1}{2}kx^2$$ 【2%】

$(10.0)(9.8)d\sin 30^{\circ} = (0.5)(340)(d)^2$

$\therefore\ d = 0.288$ (m) (or 29 cm) 【3%】

6. Two balls are vertically hung by two parallel massless ropes and contact with each other. Ball m_1 with a mass of **30 g** is pulled leftward and raised up to a height $\boldsymbol{h}$ **= 8.0 cm**. It is then released from rest to collide elastically with ball m_2 of **75 g**.

(a) (5%) Find the speed of ball m_1 right before it collides with ball m_2.

(b) (5%) Find the speeds of both balls just after the collision.

(c) (5%) What maximal height can ball m_2 reach?

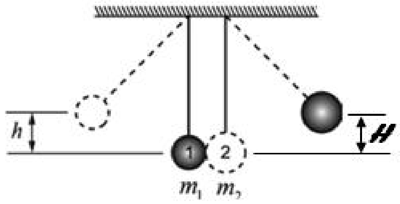
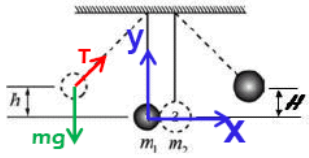


Fig. 5

[ANS]

(a) Set the coordinate as shown in the right figure.

Two forces, the rope tension **T** and the gravitational force **mg**, act on the ball m_1. [力量分析 1%]



The tension T is, though, not a conservative force, its work on the ball m_1 is zero because it is perpendicular to the motion of the ball m_1. The gravitational force is a conservative force. Therefore, the energy of the ball m_1 is conserved. [說明使用能量守恆的原因 1%]

the initial energy of the ball m_1= the final energy of the ball m_1

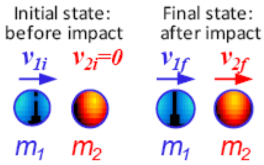
$$\rightarrow\ 0 + m_1 g h = \frac{1}{2} m_1 v_{1f} + 0\ \rightarrow\ v_{1f} = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(0.08\mathrm{m})} = 1.252\ \mathrm{m/s}$$

[完整的計算過程 2%, 最終答案 1%] (若未寫計算過程, 只有最終答案, 不給分)

(b) Before the ball m_1 is about to hit the ball m_2.

The system consisted of two balls that moving along horizontal direction and no external forces along the sliding, x, direction is acting on them.

Therefore $\sum \overrightarrow{F_x} = 0$ and the linear momentum is conserved. [說明動量守恆的原因 1%]



⇨ Initial momentum of two balls = Final momentum of two balls

⇨ $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Because this is an elastic collision, the total energy of this system is conserved.

⇨ the initial energy before the collision = the final energy after the collision

⇨ $\frac{1}{2} m_1 v_{1i}^2 + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

From these two equations:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gh}$$

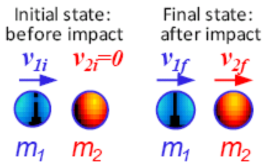
$$= \frac{(0.030\mathrm{kg}) - (0.075\mathrm{kg})}{(0.030\mathrm{kg}) + (0.075\mathrm{kg})}\sqrt{2(9.8\ \mathrm{m/s^2})(0.08\mathrm{m})} = -0.537\ \mathrm{m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + m_2}\sqrt{2gh}$$

$$= \frac{2(0.030\mathrm{kg})}{(0.030\mathrm{kg}) + (0.075\mathrm{kg})}\sqrt{2(9.8\ \mathrm{m/s^2})(0.08\mathrm{m})} = 0.715\ \mathrm{m/s}$$

[完整的計算過程 2%，最終答案 2%]
(若未寫計算過程，只有最終答案，不給分)

(c) Similar argument as the answer of (a), the initial energy of the ball m_2 is equal to the final energy of the ball m_2 [說明使用能量守恆的原因 2%]



$$\rightarrow\ \frac{1}{2} m_2 v_{2f}^2 + 0 = 0 + m_2 g H$$

$$\rightarrow\ \mathrm{H} = \frac{v_{2f}^2}{2\mathrm{g}} = \frac{0.715^2}{2\times 9.8}(\mathrm{m}) = 0.026\ \mathrm{m} = 2.6\mathrm{cm}$$

[完整的計算過程 2%, 最終答案 1%] (若未寫計算過程, 只有最終答案, 不給分) (若視 g=10 m/s², 一樣給分)

7. (10%) The block 2 (mass **1.0 kg**) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant **200 N/m**. The other end of the spring is fixed to a wall. Block 1 (mass **2.0 kg**), traveling at speed $\boldsymbol{v_1}$ **= 4.0 m/s**, collides with block 2, and then the two blocks are stick together. When the blocks momentarily stop, what distance is the spring compressed?

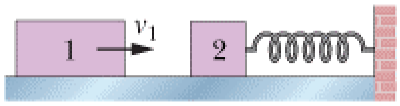


Fig. 6

[ANS]

∵When the block 1 hits the block 2, they adhere together. This is a completely inelastic collision. However, there are no other external force is acting on this block along horizontal direction. Therefore $\sum \overrightarrow{F_x} = 0$ and the linear momentum is conserved. [說明動量守恆的原因 2%]

⇨ $m_1 v_1 = (m_1 + m_2)v$

⇨ $(2.0\text{kg})(4.0\text{ m/s}) = (2.0\text{kg}+1.0\text{kg})v$

⇨ $v = 8/3$ m/s

[完整的計算過程 2%, 最終答案 1%]

After the collision, the two blocks starts to compress the spring. Since there is no non-conservative force acting on this system along horizontal direction, the total energy of this system is conserved. [說明使用能量守恆的原因 2%]

∴the initial energy of the two blocks and spring equals to their final energy.

⇨ $\frac{1}{2}(m_1 + m_2)v^2 + 0 = 0 + \frac{1}{2}kx^2$

⇨ $\frac{1}{2}(3\text{kg})(\frac{8}{3}\text{ m/s})^2 + 0 = 0 + \frac{1}{2}(200\,\frac{\text{N}}{\text{m}})x^2$

$x = (8/75)^{1/2}$ m

[完整的計算過程 2%，最終答案 1%]

8. A thin, uniform rod of mass M and length L, on an x axis with the origin at the rod's center.
   (a) (3%) What is the rotational inertia of the rod about the perpendicular rotation axis through the center?
   (b) (2%) What is the rod's rotational inertia about a new rotation axis that is perpendicular to the rod and through one end?

(a) 根據轉動轉量的定義：$I = \int x^2\,dm$ ，且 $dm = \dfrac{M}{L}dx$

則繞穿過棒子中心點垂直軸之轉動慣量為：$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\,dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \dfrac{M}{L}dx = \dfrac{1}{12}ML^2$ (3%)

(b) 根據平行軸定理：$I = I_0 + Mh^2$ ，其中h為新選定與原選定軸之距離。

則棒子繞穿過左端垂直軸轉動之轉動慣量為 $I = \dfrac{1}{12}ML^2 + M(\dfrac{L}{2})^2 = \dfrac{1}{3}ML^2$ (2%)

9. (10%) In Fig. 7, a small block of mass **50 g** slides down a frictionless surface through height ***h* = 15 cm** and then sticks to a uniform rod of mass **100 g** and length **35 cm**. The rod pivots about point O through angle θ before momentarily stopping. Find θ.

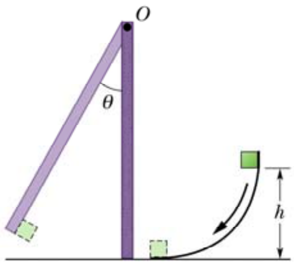


Fig. 7

block自高h處沿無摩擦表面下滑過程，在碰觸到rod前一瞬間，非保守力做功為零，故力學能守恆：

$\frac{1}{2}m_b v_i^2 + m_b g y_i = \frac{1}{2}m_b v_f^2 + m_b g y_f$ [1%]，m為block質量，下標i與f分別表示block剛下滑與碰觸rod前之狀態。選擇水平面為零位面，代入$v_i = 0$，$y_i = h$ 及$y_f = 0$，可得

$v_f = \sqrt{2gh}$。(1%)

block以此速度碰觸rod，並與之黏在一起以O點轉動，此過程中block-rod不受外力作用，故角動量守恆[1%]。假設rod長l，質量M，則：

$$\vec{r} \times \vec{p} = I\vec{\omega} \rightarrow l(mv_f)\sin\frac{\pi}{2} = I\omega \rightarrow \omega = \frac{lmv_f}{I} \text{ (2\%)}$$

I為block黏在rod上之總轉動慣量：$I = \frac{1}{3}Ml^2 + ml^2$，故$\omega = \frac{lmv_f}{I} = \frac{m\sqrt{2gh}}{(\frac{1}{3}M + m)l}$。(1%)

再次利用轉動力學能守恆，即rod-block之轉動動能轉化為rod-block之位能變化：

$$\frac{1}{2}I\omega^2 = mgl(1-\cos\theta) + Mg(\frac{l}{2})(1-\cos\theta) \text{ (2\%)}$$

代入轉動慣量I與角速度ω：

$$\frac{1}{2}(\frac{1}{3}Ml^2 + ml^2)[\frac{m\sqrt{2gh}}{(\frac{1}{3}M + m)l}]^2 = [mgl + Mg(\frac{l}{2})](1-\cos\theta)$$

$$\cos\theta = 1 - \frac{6m^2h}{(M+2m)(M+3m)l} = 1 - \frac{6 \times 0.05^2 \times 0.15}{(0.1 + 2 \times 0.05)(0.1 + 3 \times 0.05)(0.35)} = 0.87 \text{ (2\%)}$$

$\theta \simeq 29°$

10. (10%) Let the disk with radius **0.20 m** in Fig. 8 starts from rest at time ***t* = 0** and also let the tension in the massless cord be **6.0 N** and the magnitude of angular acceleration of the disk be **24 rad/s^2**. Using work-kinetic energy theorem to find the rotational kinetic energy of the disk at ***t* = 2.5 s**?

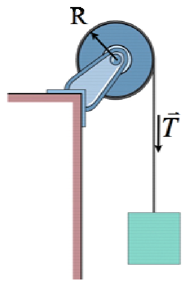


Fig.8

sol:

根據功-動能原理：$W = \Delta E_k = E_{kf} - E_{ki}$

由於disk初始為靜止($E_{ki} = 0$)，故$E_{kf} = W$ ---(1)，恰為disk在t=2.5 s之轉動動能。(2%)

繩上之張力$\vec{T}$ 作用於disk上，所對應之轉動矩$\vec{\tau} = \vec{R} \times \vec{T} = RT(\otimes)$ 。(1%)

故所做之功$W = \int_{\theta_i}^{\theta_f} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_i}^{\theta_f} (TR\otimes) \cdot (d\theta\otimes) = TR(\theta_f - \theta_i)$ ---(2) (2%)

因作用於disk之轉動矩為定值(1%)，故disk轉動為等角加速度運動：

$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ (1%)

初始disk靜止，故$\omega_i = 0 \rightarrow \theta_f - \theta_i = \frac{1}{2}\alpha t^2$ ---(3) (1%)

綜合(1), (2), (3)：$E_{kf} = TR(\frac{1}{2}\alpha t^2) = (6.0)(0.20)(\frac{1}{2} \times 24 \times 2.5^2) = 90(J)$

(2%)