國立中山大學 98 學年度普通物理(二)期末考 99.06.19

作答時，請由答題紙第六列開始填寫，並標明題號。考試時間爲 13：00~15：30，14：00 以後方可交卷。
作答時，請詳列相關公式及計算過程，並記得寫上單位，建議以 M.K.S 制表示。
【$\mu_0 = 4\pi\times10^{-7}$ $C^2/N\cdot m^2$，$\varepsilon_0 = 8.85\times10^{-12}$ T·m/A】

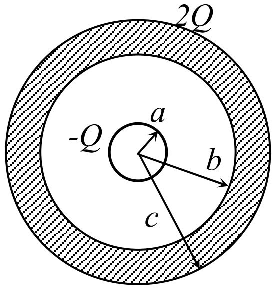


Fig. 1

1. **(12%)** A thin conducting sphere shell of radius a carries a net negative charge $-Q$ uniformly distributed throughout its surface. A conducting spherical hollow shell of inner radius b and outer radius c is concentric with the spherical shell and carries a net charge $2Q$. Using Gauss's law, find the electric field in the regions where the radius r satisfies (a) $r < a$, (b) $a < r < b$, (c) $b < r < c$, (d) $r > c$.

Applying Gauss's law, we could find the Gaussian surface where electric field is constant.

(a) For $r < a$,

The electric field must be zero inside the conducting spherical shell. $E = 0$

(b) For $a < r < b$,

$$\oint EdA = E\oint dA = E(4\pi r^2) = \frac{-Q}{\varepsilon_0}$$

$$E = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (or\ -k_e\frac{Q}{r^2})$$

(c) For $b < r < c$,

The electric field must be zero inside the conducting spherical shell. $E = 0$

(d) For $r > c$,

$$\oint EdA = E\oint dA = E(4\pi r^2) = \frac{2Q - Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (or\ k_e\frac{Q}{r^2})$$

【列出 Gauss's Law 給 4 分，算出正確結果每一小題給 2 分】

2. **(8%)** A parallel-plate capacitor of plate area A is filled with two dielectrics as in Fig. 2 and Fig. 3. Find the capacitance in both cases.

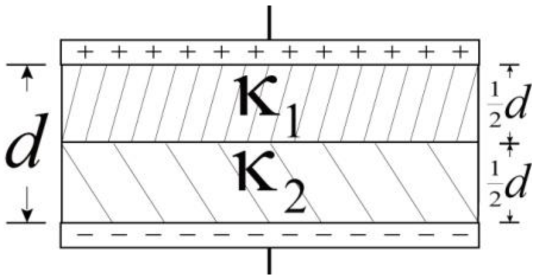


Fig. 2

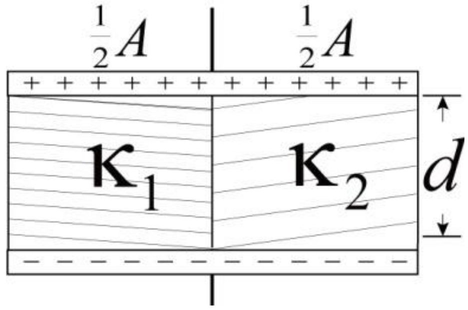


Fig. 3

【了解平行板之電容爲 $C=\frac{\varepsilon_0 A}{d}$ 給 2 分】

(i) *in* Fig. 2, $C_1=\frac{\kappa_1\varepsilon_0 A}{d/2}, C_2=\frac{\kappa_2\varepsilon_0 A}{d/2}$ (2%)

$$\therefore \frac{1}{C}=\frac{1}{C_1}-\frac{1}{C_2}=\frac{d}{2\kappa_1\varepsilon_0 A}+\frac{d}{2\kappa_2\varepsilon_0 A}=\frac{d}{2\varepsilon_0 A}\left(\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right)=\frac{d}{2\varepsilon_0 A}\left(\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2}\right)$$

$\therefore C=\frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right)$ (1%)

(ii) *in* Fig. 3, $C_1=\frac{\kappa_1\varepsilon_0\left(\frac{A}{2}\right)}{d}, C_2=\frac{\kappa_2\varepsilon_0\left(\frac{A}{2}\right)}{d}$ (2%)

$\therefore C=C_1+C_2=\frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1+\kappa_2}{2}\right)$ (1%)

3. Fig. 4 shows a cross section of a long cylindrical conductor of radius $a = 4.00$ *cm* containing a long cylindrical hole of radius $b = 1.50$ *cm*. The central axes of the cylinder and hole are parallel and are distance $d = 2.00$ *cm* apart; current $i = 5.25$ *A* is uniformly distributed over the grey area.
   (a) **(6%)** What is the magnitude of the magnetic field at the center of the hole?
   (b) **(6%)** Discuss the two special cases $b = 0$ and $d = 0$.

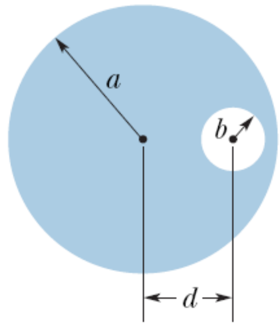


Fig. 4

(a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current *i* which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2} \qquad (2\%)$$

at a distance *r* from its axis, inside the cylinder. Here *R* is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi a^2 - \pi b^2}$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = J\pi a^2 = \frac{ia^2}{a^2 - b^2} \qquad (1\%)$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2 (a^2 - b^2)} = \frac{\mu_0 i r_1}{2\pi (a^2 - b^2)}$$

The current in the cylinder that fills the hole is

$$I_2 = J\pi b^2 = \frac{ib^2}{a^2 - b^2} \qquad (1\%)$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2 (a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi (a^2 - b^2)}$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})\left(5.25\mathrm{A}\right)(0.0200\mathrm{m})}{2\pi[(0.0400\mathrm{m})^2 - (0.0150\mathrm{m})^2]} = 1.53\times10^{-5}\,\mathrm{T} \qquad (2\%)$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2} \qquad (3\%)$$

This correctly gives the field of a solid cylinder carrying a uniform current *i*, at a point inside the cylinder a distance *d* from the axis.

If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current. (3%)

4. **(8%)** Fig. 5 shows a circular region of radius $R = 4.00\ cm$ in which a <u>uniform</u> electric flux is directed out of the plane of the page. The total electric flux through the region is given by $\Phi_E = 4.00\times10^{-3}\ t(V\cdot m/s)$, where t is in seconds. What is the magnitude of the magnetic field that is induced at radial distances (a) $2.00\ cm$ and (b) $6.00\ cm$?

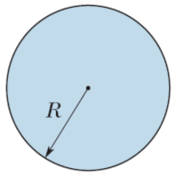


Fig. 5

$$(a)\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

$$B\cdot 2\pi r = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}\frac{\pi r^2}{\pi R^2}$$

$$B = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}\frac{r}{2\pi R^2} = \mu_0\varepsilon_0(4.00\times10^{-3})\frac{r}{2\pi R^2} \quad (3\%)$$

$$= 8.85\times10^{-12}\cdot 4\pi\times10^{-7}\cdot 4.00\times10^{-3}\frac{0.02}{2\pi(0.04)^2} = 8.85\times10^{-20}\,T \quad (1\%)$$

$$(b)B\cdot 2\pi r = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0(4.00\times10^{-3})$$

$$B = \mu_0\varepsilon_0(4.00\times10^{-3})\frac{1}{2\pi r} \quad (3\%)$$

$$= 8.85\times10^{-12}\cdot 4\pi\times10^{-7}\cdot 4.00\times10^{-3}\frac{1}{2\pi(0.06)} = 1.18\times10^{-19}\ T \quad (1\%)$$

5. **(6%)** The magnitude of the electric field between the two circular parallel plates of radius $R = 10\ m$ in Fig. 6 is $E = (4.0\times10^5) - (3.0\times 10^4)t$, with E in volts per meter and t in seconds. At $t = 0$, E is upward. For $t \geqq 0$, (a) what is the displacement current between the plates (write down the magnitude and direction (up or down)) (b) what is the direction of the induced magnetic field (clockwise or counterclockwise) in the Fig. 6 (viewing from the top)?

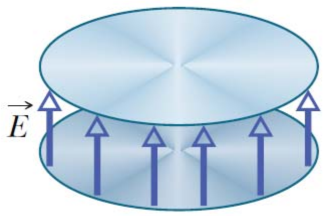


Fig. 6

(a)

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{dEA}{dt} \quad (2\%)$$

$$= \varepsilon_0 A\frac{dE}{dt} = 8.85\times10^{-12}\cdot\pi(10)^2\cdot(-3.0\times10^4) = -8.34\times10^{-5}\ A$$

8.34×10^{-5} A (1%) and down (1%)

(b)

Clockwise (2%)

6. **(20%)** A plane electromagnetic wave, with wavelength *3 m*, travels in free space in the + *x* direction with its electric vector $\vec{E}$, of amplitude *300 V/m*, directed along the *y* axis. (a) What is the frequency of the wave? (b) What is direction and amplitude of the magnetic field associated with wave? (c) If $E = E_m \sin(kx - \omega t)$, what are the values of *k* and ω? (d) Find the intensity of the wave. (e) If the wave falls on a perfectly absorbing sheet of area $2.0\ m^2$, at what rate would momentum be delivered to the sheet and what is the radiation pressure exerted on the sheet?

已知 波長$\lambda = 3\ m$, 電場振幅 $E_m = 300\ V/m$ $\vec{E}(x,t) = \hat{y}E_m \sin(kx - \omega t)\ \ V/m$

(a)電磁波的頻率f爲 $f = \dfrac{c}{\lambda}$ (2%) $= \dfrac{3\times10^8 (m/s)}{3(m)}$ (2%) $= 10^8\ Hz$(or $1/s$)

(b)由 Poynting vector $\vec{S} = \dfrac{1}{\mu_o}\vec{E}\times\vec{B}$

已知$\vec{E} = \hat{y}E_y$, $\vec{S} = \hat{x}S_x$, 設$\vec{B} = \hat{x}B_x + \hat{y}B_y + \hat{z}B_z$代入上式

$$\hat{x}S_x = \frac{1}{\mu_o}(\hat{y}E_y)\times(\hat{x}B_x + \hat{y}B_y + \hat{z}B_z)$$

$$= \frac{E_y}{\mu_o}(-\hat{z}B_x + \hat{x}B_z) = -\hat{z}\frac{E_yB_x}{\mu_o} + \hat{x}\frac{E_yB_z}{\mu_o}$$

$$0 = \frac{E_yB_x}{\mu_o}, \qquad \therefore \quad B_x = 0$$

$$S_x = \frac{E_yB_z}{\mu_o}, \qquad \therefore \quad \vec{B} = \hat{z}B_z$$

故磁場的方向爲正 *z* 方向. (2%)【直接寫出磁場方向亦給分】

由$\dfrac{E_m}{B_m} = c$得磁場的振幅爲 $B_m = \dfrac{E_m}{c} = \dfrac{300(V/m)}{3\times10^8(m/s)}$ (2%) $= 10^{-6}\ T$

(c) $\vec{E}(x,t) = \hat{y}E_m \sin(kx - \omega t)$

波數 $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi(rad)}{3(m)}$ (2%) $= 2.09\ rad/m$

角頻率 $\omega = 2\pi f = (2\pi\ rad)(10^8\ Hz)$ (2%) $= 6.28\times10^8\ rad/s$

(d)波的強度爲

$$I = \frac{E_m^2}{2c\mu_0} = \frac{(300\ V/m)^2}{2(3\times10^8\ m/s)(4\pi\times10^{-7}\ T\cdot m/A)} \text{ (4\%) } = 119\ W/m^2$$

(e)完全吸收面的輻射壓爲 $P_r = \dfrac{I}{c} = \dfrac{119\ (W/m^2)}{3\times10^8(m/s)}$ (2%) $= 3.97\times10^{-7}\ N/m^2$

波傳送至吸收面的動量變化率爲 $\dfrac{dP}{dt} = F = P_rA = (3.97\times10^{-7}\ N/m^2)(2\ m^2)$ (2%) $= 7.94\times10^{-7}\ N$

7. **(10%)** A piece of transparent material having an index of refraction n is cut into the shape of a wedge as shown in Fig. 7. The wedge is placed in the air, and angle of the wedge is small. Monochromatic light of wavelength λ is normally incident from above, and viewed from above. Let h represent the height of the wedge and ℓ its width. Find the positions (x = ?)of bright fringes and dark fringes.

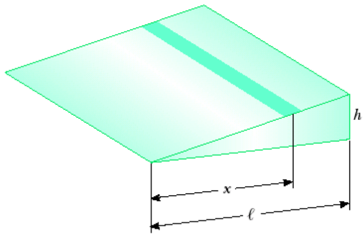
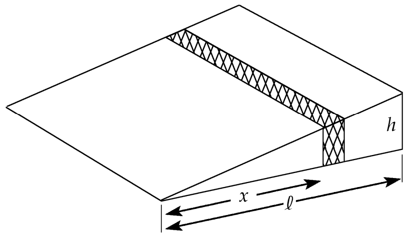


Fig. 7

Bright fringes occur when $2t = \frac{\lambda}{n}\left(m + \frac{1}{2}\right)$ $m = 0, 1, 2, \cdots$ (2%)

and dark fringes occur when $2t = \left(\frac{\lambda}{n}\right)m$ $m = 0, 1, 2, \cdots$ (2%)

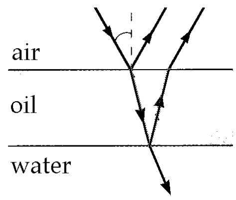
The thickness of the film at x is $t = \left(\frac{h}{\ell}\right)x$.



Therefore, $\boxed{x_{\text{bright}} = \frac{\lambda\ell}{2hn}\left(m + \frac{1}{2}\right)}$ $m = 0, 1, 2, \cdots$(3%) and $\boxed{x_{\text{dark}} = \frac{\lambda\ell m}{2hn}}$ $m = 0, 1, 2, \cdots$ (3%).

8. **(10%)** An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is *280 nm* thick. Find (a) the color of the light in the visible spectrum most strongly reflected and (b) the color of the light in the spectrum most strongly transmitted. Explain your reasoning.

(a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have



$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m + (1/2)} = \frac{2(1.45)(280\text{ nm})}{m + (1/2)}.$$ (1%)

Substituting for m gives: $m = 0$, $\lambda_0 = 1\,620\text{ nm}$ (infrared) (1%)

$m = 1$, $\lambda_1 = 541\text{ nm}$ (green) (1%)

$m = 2$, $\lambda_2 = 325\text{ nm}$ (ultraviolet). (1%)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$. (1%)【只寫出此波長爲可見光波段亦給分】

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda \quad \text{or} \quad \lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}.$$ (1%)

Substituting for m gives: $m = 1$, $\lambda_1 = 812\text{ nm}$ (near infrared) (1%)

$m = 2$, $\lambda_2 = 406\text{ nm}$ (violet) (1%)

$m = 3$, $\lambda_3 = 271\text{ nm}$ (ultraviolet) (1%)

Of these, the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is

$\boxed{\text{violet}}$. (1%)【只寫出此波長爲可見光波段亦給分】

9. **(10%)** A single slit *1.00 mm* wide is illuminated by light of wavelength *5890 Å*, (a) what is the angle of the first minimum to one side of the central maximum (b) find the ratio of first secondary maximum intensity to the intensity of central maximum.

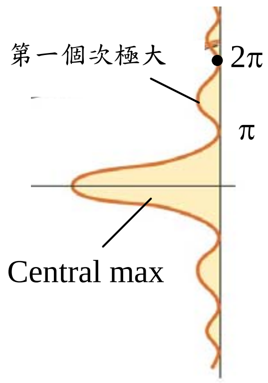
(a) For minimum diffraction $a\sin\theta = m\lambda$, $m = 1, 2, 3, \cdots$ (3%)

At first minimum, $m = 1$ $\quad \therefore \theta = \sin^{-1}\frac{\lambda}{a} = \sin^{-1}\frac{5890\,\overset{0}{\mathrm{A}}}{10^{-3}m} \cong 5.89\times 10^{-4}\,rad$ (2%)

(b) For a single slit diffraction intensity at any angle θ

$$\begin{cases} I_\theta = I_m(\frac{\sin\alpha}{\alpha})^2 \\ \alpha = \frac{\pi a}{\lambda}(\sin\theta) \end{cases}$$

I_m = central maximum (2%)



minimum intensity occur at $\alpha = m\pi$, $m = 1, 2, 3, \cdots$

次極大(secondary maximum)大約介於二個 minimum 之中央

即 $\alpha \cong (m+\frac{1}{2})\pi,\ m = 1, 2, 3, \cdots$ (2%)

∴第一個次極大(first secondary maximum)與 I_m 比值

$$\frac{I_\theta}{I_m} = [\frac{\sin(m+1/2)\pi}{(m+1/2)\pi}]^2 \Rightarrow \frac{1}{[(1+\frac{1}{2})\pi]^2} = \frac{4}{9\pi^2} = 0.045$$ (1%)

10. **(5%)** A diffraction grating has 10^4 rulings, uniformly spaced over *2.54 cm*, It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely lines of wavelength *5890* Å and *5895.9* Å. Calculate the angular separation between these two lines in the first order maximum.

光柵繞射最大強度之條件 $\quad d\sin\theta = m\lambda, \quad m = 1, 2, 3, \cdots$

first maximum, m = 1 $\quad \therefore d\sin\theta = \lambda$ (1) (1%)

$\because d = \frac{2.54cm}{10^4} = 25400\,\mathrm{\mathring{A}}$ (2) (1%)

由(1) $\quad \theta = \sin^{-1}\frac{5890\,\mathrm{\mathring{A}}}{25400\,\mathrm{\mathring{A}}} = \sin^{-1}0.232 \cong 13.3°$

$\because d\cos\theta\,\Delta\theta = m\Delta\lambda, \quad m = 1$ (1%)

鈉光二黃線λ_1及λ_2 angular separation 爲

$$\Delta\theta = \frac{\Delta\lambda}{d\cos\theta} \cong \frac{(5895.9-5890)\,\mathrm{\mathring{A}}}{25400\,\mathrm{\mathring{A}}\times\cos 13.3°} = 2.4\mathrm{x}10^{-4}\ \mathrm{rad}$$ (2%)

11. **(5%)** An x-ray beam is incident on a NaCl crystal, at $\theta = 30°$ to a certain family of reflecting plane of spacing $d = 3.00$ Å, first order of Bragg reflection is observed. What is the wavelength of x –ray?

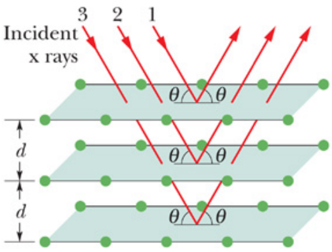


Fig. 8

$\because 2d\sin\theta = m\lambda$, $m = 1$ (3%)

$\therefore \lambda = 2d\sin\theta = 2\times3\times\sin30° = 3$ Å (2%)